Conservation of Linear Momentum in Chemical Reactions

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Introduction

The purpose of this poster is to prove the law of linear momentum conservation but to do that the following laws must be understood and assumed to be true:

Closed System - A closed system is one that is isolated from its surroundings and does not interact in any way with the outside system.

Law of Conservation of Mass - The total mass of reactants prior to the reaction will be equivalent to the total mass of the products post reaction.

Law of Conservation of Energy - The total amount of energy in a system remains constant.

Galileo Galilei's Principle of Relativity- Inertial reference frames all mechanical processes are the same.

Law of Conservation of Momentum - Total amount of momentum will remain constant at the beginning of a reaction and at the end.

Momentum = ρ = mass X velocity

Objectives

Our objective for this research project was to prove the law of conservation of linear momentum in a chemical reaction using the law of conservation of mass, conservation of energy, and Galileo Galilei' s principle of relativity.

The Reaction

A + BC → B + AC

Let the reaction be a single replacement chemical reaction where BC is a compound consisting of atoms B and C. The bond between BC is broken and atom C binds to atom A to create the products of a B atom and AC compound.

Let:
- •m_A be the mass of atom A.
- •v_i be the initial velocity of atoms and molecules just before the reaction.
- •m_B be the mass of atom B.
- •v_f be the final velocity of atoms and molecules just after the reaction.
- •m_C be the mass of atom C.

Proof

Using the two given laws about chemical reactions, the law of conservation of energy in one inertial reference frame can be written as (Eq. 1)

$$\frac{1}{2}m_A(v_{iA})^2+\frac{1}{2}(m_B+m_C){v_{iBC}}^2=\frac{1}{2}m_B(v_{fB})^2+\frac{1}{2}(m_A+m_C){v_{fAC}}^2+\Delta E$$

Law of conservation of energy in the second inertial reference frame, using Galileo's principle of relativity can be written as (Eq. 2)

$$\frac{1}{2}m_A(v_{iA}-V)^2+\frac{1}{2}(m_B+m_C)(v_{iBC}-V)^2$$
$$=\frac{1}{2}m_B(v_{fB}-V)^2+\frac{1}{2}(m_A+m_C)(v_{fAC}-V)^2+\Delta E$$

Now doing algebra as usual, expansion of the squared quantities (Eq. 2)

$$\frac{1}{2}m_A({v_{iA}}^2-2v_{iA}V+V^2)+\frac{1}{2}(m_B+m_C)({v_{iBC}}^2-2v_{iBC}V+V^2)$$
$$=\frac{1}{2}m_B({v_{fB}}^2-2v_{fB}V+V^2)+\frac{1}{2}(m_A+m_C)({v_{fAC}}^2-2v_{fAC}V+V^2)+\Delta E$$

(Eq. 2)*2 and distribution of the masses (Eq. 2)

$$m_A(v_{iA})^2-2m_A(v_{iA})(V)+m_A(V^2)+m_B(v_{iBC})^2-2m_B(v_{iBC})(V)+m_B(V^2)$$
$$+m_C(v_{iBC})^2-2m_C(v_{iBC})(V)+m_C(V^2)$$
$$=m_B(v_{fB})^2-2m_B(v_{fB})(V)+m_B(V^2)+m_A(v_{fAC})^2-2m_A(v_{fAC})(V)$$
$$+m_A(V^2)+m_C(v_{fAC})^2-2m_C(v_{fAC})(V)+m_C(V^2)+2\Delta E$$

Cancellation of the terms with V^2 (Eq. 2)

$$m_A(v_{iA})^2-2m_A(v_{iA})(V)+m_B(v_{iBC})^2-2m_B(v_{iBC})(V)+m_C(v_{iBC})^2$$
$$-2m_C(v_{iBC})(V)$$
$$=m_B(v_{fB})^2-2m_B(v_{fB})(V)+m_A(v_{fAC})^2-2m_A(v_{fAC})(V)+m_C(v_{fAC})^2$$
$$-2m_C(v_{fAC})(V)+2\Delta E$$

Rearrangement of terms (Eq. 2)

$$m_A(v_{iA})^2-2m_A(v_{iA})(V)+(m_B+m_C)(v_{iBC})^2-2(m_B+m_C)(v_{iBC})(V)$$
$$=m_B(v_{fB})^2-2m_B(v_{fB})(V)+(m_A+m_C)(v_{fAC})^2-2(m_A+m_C)(v_{fAC})(V)$$
$$+2\Delta E$$

(Eq. 1)*2

$$m_A(v_{iA})^2+(m_B+m_C){v_{iBC}}^2=m_B(v_{fB})^2+(m_A+m_C){v_{fAC}}^2+2\Delta E$$

(Eq. 2) – (Eq. 1)

$$-2m_A(v_{iA})(V)-2(m_B+m_C)(v_{iBC})(V)$$
$$=-2m_B(v_{fB})(V)-2(m_A+m_C)(v_{fAC})(V)$$

Cancellation of (-2) and V (Eq. 3)

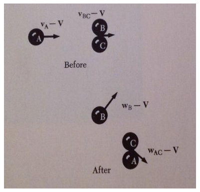
$$m_A(v_{iA})+(m_B+m_C)(v_{iBC})=m_B(v_{fB})+(m_A+m_C)(v_{fAC})$$

Results

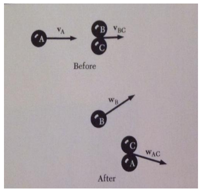
Using a closed system, we proved the conservation of linear momentum in chemical reactions. The total mass, energy, and linear momentum of the system remained constant under the closed system as there was no external net force acting upon it. The momentum equation was derived from a combination of the conservation of mass and conservation of energy and yielded the final equation:

$$m_A(v_{iA})+(m_B+m_C)(v_{iBC})=m_B(v_{fB})+(m_A+m_C)(v_{fAC})$$

This means that the sum of the initial atom's and molecule's momentum will equal the total final momentum of the system. Without utilizing Galileo Galilei's principle of relativity, the conservation of linear momentum in a chemical reaction would be impossible to prove. It must be known that all mechanical processes are the same in all inertial reference frames, which is why linear momentum can be proven. The principle of relativity confirms that for all chemical reactions, molecules and atoms will participate in the conservation of linear momentum.



First Inertial Reference Frame



Second Inertial Reference Frame

References

Famous Scientists. *Science Blog RSS*. N.p., Sept. 2013. Web. 30 Oct. 2013.
Antoine-Laurent Lavoisier. *Homepage of the Chemical Heritage Foundation*. N.p., n.d. Web. 30 Oct. 2013.
Antoine Lavoisier. *Antoine Lavoisier*. N.p., 25 Sept. 2001. Web. 30 Oct. 2013.
Antoine Lavoisier. *Science Blog RSS*. N.p., Sept. 2013. Web. 30 Oct. 2013.
Cleveland State University *Honors University Physics 1- 243* Lecture Notes
Kittel, Charles, Walter D. Knight, and Malvin A. Ruderman. "Frames of Reference: Galilean
Transformation." *Berkeley Physics Course.* Vol. 1. New York: McGraw-Hill, 1965. 118-123. Print.
Halliday, David, Robert Resnick, and Jearl Walker. *Fundamentals of Physics.* 10th ed.
Hoboken: John Wiley & Sons, 2011. Online.