The Mathematics Behind Card Counting

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Introduction:

Counting cards is a technique used in black jack to help increase the probability of winning big. Using this technique allows players to analyze the likelihood of receiving certain cards from the dealer, which helps them determine what decisions to make in the game.

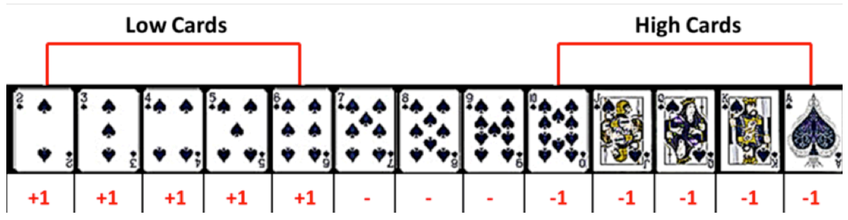
Influences on the Scientific Community:

The most influential part of card counting that impacted the scientific community was the discovery of the Kelly Criterion formula

- It is an algorithm for maximizing bets and has been loosely used for mainstream investment theory.

How Counting Cards Works:

- Assign a value of +1 to cards 2-6, 0 to cards 7-9, and -1 to cards 10-A
- As each card is dealt, keep a running count for all the cards on the table
- Use the formula: *True Count= Running Count/ Decks Remaining*
- As the true count rises, the player should increase their bets because the odds are in their favor



Rules for Calculations:

1. Original hand is dealt
2. The dealer and player have a maximum of one hit
3. The dealer only hits if they have less than a 17
4. If there is a tie, the house win

Expected Return:

We can calculate the amount you are expected to win using probability, which will reflect why counting cards can help you win money, because it corresponds with the count of the deck in terms of the larger count of deck equals a larger payout.

*Expected Return = P(losing)(-1) + P(winning)(+1)

Expected Payout:

Expected Payout = Payout x P(win) – Bet x P(loss)

-Payout : amount of money the casino gives if you win on top of the money you get back

*Note: The following calculations find expected payout when the dealer takes a hit and the player stays in order to show how expected return corresponds with the count of the deck.

X = first hit

Y = second hit

Probability That The Dealer Wins: Card Situation - Player (10,7) Dealer (6,X)

Scenario 1 : First Deal of the Deck

- Case 1-

  P(X=A) = 4/49
- Case 2-

  P(X=2 and Y=9,10,A) = 4/49 x 23/48 = 92/2352

  P(X=3 and Y=8,9,10,A) = 4/49 x 27/48 = 108/2352

  P(X=4 and Y=7,8,9,10,A) = 4/49 x 30/48 = 120/2352

  P(X=5 and Y=6,7,8,9,10) = 4/49 x 29/48 = 116/2352

  P(X=6 and Y=5,6,7,8,9) = 3/49 x 17/48 = 51/2352

  P(X=7 and Y=4,5,6,7,8) = 3/49 x 17/48 = 51/2352

  P(X=8 and Y=3,4,5,6,7) = 4/49 x 18/48 = 72/2352

  P(X=9 and Y=2,3,4,5,6) = 4/49 x 19/48 = 76/2352

  P(X=10 and Y=A,2,3,4,5) = 15/49 x 20/49 = 300/2352

Total = 4/49 + 986/2352 = .5008503402

P(dealer wins when player stands) = .5008503402

P(player wins when they stand) = .4991496598

Expected Return:

.5008503402(-1) + .4991496598(+1) = -.0017006804

Scenario 2 : 40 cards left in deck for this hand an count of the deck is -5 after two hands where both dealer and player took a hit.

A	K	Q	J	10	9	8	7	6	5	4	3	2	
4	3	2	3	3	2	1	2	4	4	4	4	4	
	-1	-2	-1	-1	+0	+0	+0						-5

- Case 1-

  P(X=A) = 4/37
- Case 2-

  P(X=2 and Y=9,10,A) = 4/37 x 16/36 = 64/1332

  P(X=3 and Y=8,9,10,A) = 4/37 x 17/36 = 68/1332

  P(X=4 and Y=7,8,9,10,A) = 4/37 x 18/36 = 72/1332

  P(X=5 and Y=6,7,8,9,10) = 4/37 x 17/36 = 68/1332

  P(X=6 and Y=5,6,7,8,9) = 3/37 x 10/36 = 30/1332

  P(X=7 and Y=4,5,6,7,8) = 1/37 x 12/36 = 12/1332

  P(X=8 and Y=3,4,5,6,7) = 1/37 x 16/36 = 16/1332

  P(X=9 and Y=2,3,4,5,6) = 2/37 x 19/36 = 38/1332

  P(X=10 and Y=A,2,3,4,5) = 3/37 x 20/36 = 60/1332

  P(X=J and Y=A,2,3,4,5) = 3/37 x 20/36 = 60/1332

  P(X=Q and Y=A,2,3,4,5) = 2/37 x 20/36 = 40/1332

  P(X=K and Y=A,2,3,4,5) = 3/37 x 20/36 = 60/1332

Total = 4/37 + 588/1332 = .54955

P(dealer wins when player stands) = .54955

P(player wins) = .45045

Expected Return:

.54955(-1) + .45045(+1) = -.0991

- This means the player is expected to lose about 10 cents on this hand

Scenario 3 : 40 cards left in the deck for this hand and the count of the deck is 10 after two hands where players both took a hit already happened.

A	K	Q	J	10	9	8	7	6	5	4	3	2	
4	4	4	4	4	3	3	4	2	1	3	3	1	
					+0	+0		+2	+3	+1	+1	+3	10

- Case 1-

  P(X=A) = 4/37
- Case 2-

  P(X=2 and Y=9,10,A) = 1/37 x 22/36 = 22/1332

  P(X=3 and Y=8,9,10,A) = 3/37 x 25/36 = 75/1332

  P(X=4 and Y=7,8,9,10,A) = 3/37 x 28/36 = 84/1332

  P(X=5 and Y=6,7,8,9,10) = 1/37 x 25/36 = 25/1332

  P(X=6 and Y=5,6,7,8,9) = 1/37 x 10/36 = 10/1332

  P(X=7 and Y=4,5,6,7,8) = 3/37 x 10/36 = 30/1332

  P(X=8 and Y=3,4,5,6,7) = 3/37 x 11/36 = 33/1332

  P(X=9 and Y=2,3,4,5,6) = 3/37 x 9/36 = 27/1332

  P(X=10 and Y=A,2,3,4,5) = 3/37 x 12/36 = 36/1332

  P(X=J and Y=A,2,3,4,5) = 4/37 x 12/36 = 48/1332

  P(X=Q and Y=A,2,3,4,5) = 4/37 x 12/36 = 48/1332

  P(X=K and Y=A,2,3,4,5) = 4/37 x 12/36 = 48/1332

Total = 4/37 + 486/1332 = .472973

P(dealer wins when player stands) = .472973

P(player wins) = .527027

Expected Return:

.472973(-1) + .527027(+1) = .054054

- This means the player is expected to win about 5 cents on this hand

Future Direction:

Now that we have derived the mathematics behind "counting cards" and proven an edge can precisely be determined, how can this information be useful in a realm beyond gambling? How about portfolio allocating for investment risks!

-The *Tiburon/Kelly Formula Variance* is a formula used to calculate position sizing for portfolios:

$$Ts = \frac{\frac{bp-q}{b}}{\sum\frac{bp-q}{b}} \leq 10\,\%$$

Ts	Tiburon position sizing (cannot exceed 10% of Tiburon portfolio at market)
b	net odds received on the wager
p	probability of winning
q	probability of losing
∑	Summation of Kelly allocations across our portfolio

-An investment professional must first identify positions that meet their criteria, then sizing the position is just a function of the manager's edge in the trade and the odds of the favorable outcome

-By using the Tiburon/Kelly Formula Variance, investors can accurately size positions and maximize profitability as well as reducing risks