Expansion of "The Art Gallery Theorem"

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Background

Victor Klee (1973)
Posed the question: How many guards are sufficient or necessary to cover the interior of an n-wall art gallery?

Vasek Chvátal (1975)
First mathematician to find a solution. His solution was that n/3 guards are always sufficient and occasionally necessary to guard a polygon with n vertices.

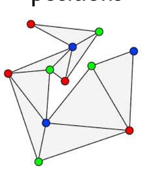
Steve Fisk (1978)
Another mathematician who came up with a much simpler and more elegant proof, with the same conclusion.

Fisk's Proof

Question: How many guards are sufficient or necessary in order to guard all areas within an n sided polygon?

Solution:

1. Triangulate the polygon, without adding extra vertices
2. Perform a 3-coloring of the vertices, where each vertex of any given triangle is a different color
3. Indicate color with least amount of vertices, thus finding the most amount of guards necessary and their positions



$A + B + C = N$

$3C^* <= N$

$C^* <= N/3$

Expanding Into Three Dimensions

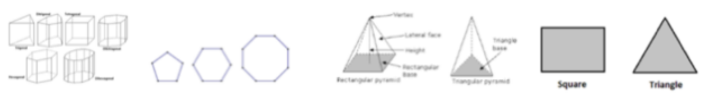
Three Dimensional Projections

Question:

How many guards are sufficient or necessary to see all areas within a polyhedron, instead of a polygon?

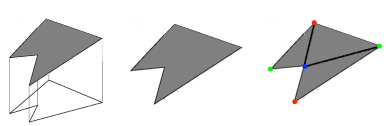
Our Solution:

In order to apply the Art Gallery Theorem to certain polyhedrons we took the 3D graph and projected it onto its corresponding 2D plane. We applied this method specifically to prisms and pyramids, using their bases as the 2D projection. Once the projection was found, we used Fisk's proof to complete the problem.



Results

Our attempt at expanding the Art Gallery Theorem to 3D was successful because we were able to simplify certain polyhedron into two dimensions. Essentially, the bases of the prisms and pyramids can be seen as the "gallery floor" which results in the outline of the walls needed to be seen by the guards. After applying Fisk's method to find the sufficient and necessary amount of guards, it is clear that the 3D projection method successfully satisfies the Art Gallery Theorem for pyramids and prisms.



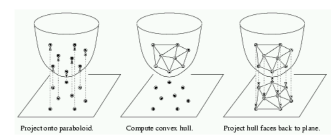
$B + G + R = N \longrightarrow 3B <= N$

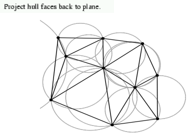
$B <= N/3 \longrightarrow 1 <= 5/3$

Shows that N/3 guards are sufficient

Future Direction

Delaunay Triangulation
Three dimensional projections can successful determine the number of guards needed to see each vertex of the base, thus successfully completing the Art Gallery Theorem. However this process doesn't take into consideration the full interior of the polyhedron. In order to triangulate the interior of the polyhedron we looked to Delaunay Triangulation. While this form of triangulation can be used during triangulation of 3D objects, it is not ideal because a tetrahedron is determined from a set of points, rather than starting with a polyhedron and forming a triangulation.





Director of COF Math at KSU:
Jenya Soprunova
Source of Interest in this topic:
Presentation on the Art Gallery Theorem given by Isaac Defrain
Sponsors:
Choose Ohio First: Success In Math
Kent State University Math Department

Sources:
http://cs.smith.edu/~orourke/books/ArtGalleryTheorems/Art_Gallery_Full_Book.pdf
http://giovanniviglietta.com/slides/carleton.pdf