Queueing Theory Applied to Organ Transplant Waiting Lists

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Abstract

Queueing theory is the mathematical study of waiting in lines, or queues. By taking a differential approach to model the wait time for an organ transplant, we can efficiently allocate organ donors to those who need them. The most involved case can be simplified, yet still effective. In this case, only O- blood type donors and O- blood type patients are being considered. Only the rate of donors available, rate of death, and rate of patients coming in are being modeled.

Introduction

- A big problem in the medical field, specifically with kidneys, hearts, livers, and other organs is the need for transplants. This research explores the factors necessary for finding donors for patients in a first come first transplanted basis.
- In this specific case, we are looking at queueing theory related to organ transplant waiting time. We are disregarding specific elements like blood type and priority for simplicity. We are only looking at O- blood donors and our only source of donor is death.

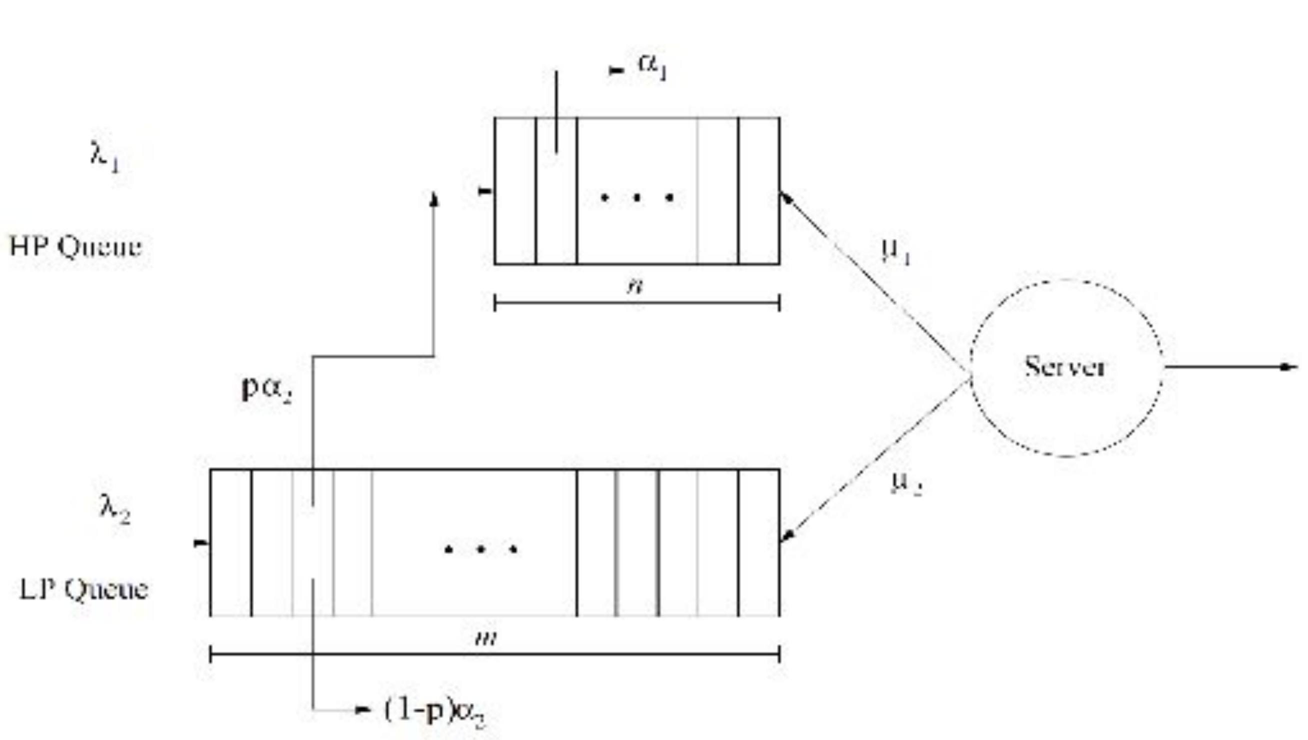
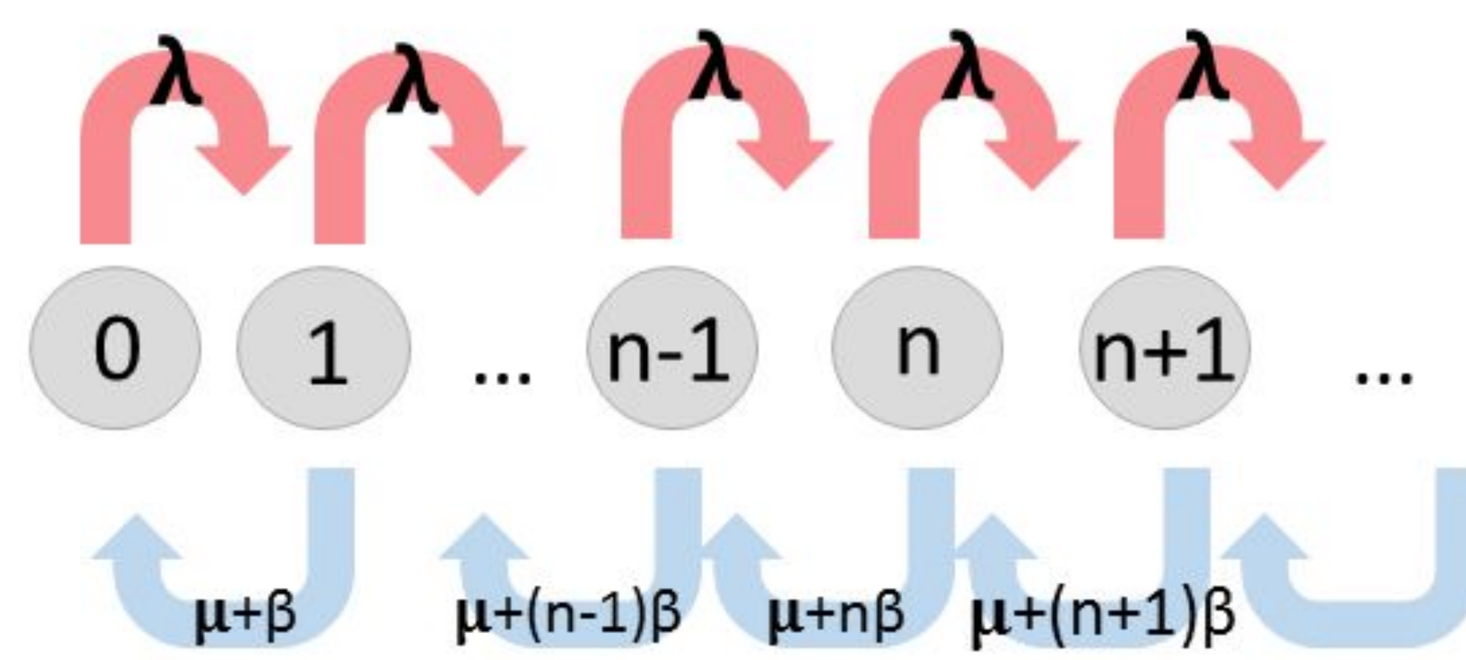


Figure 1: Proposed Queueing Model

Methods



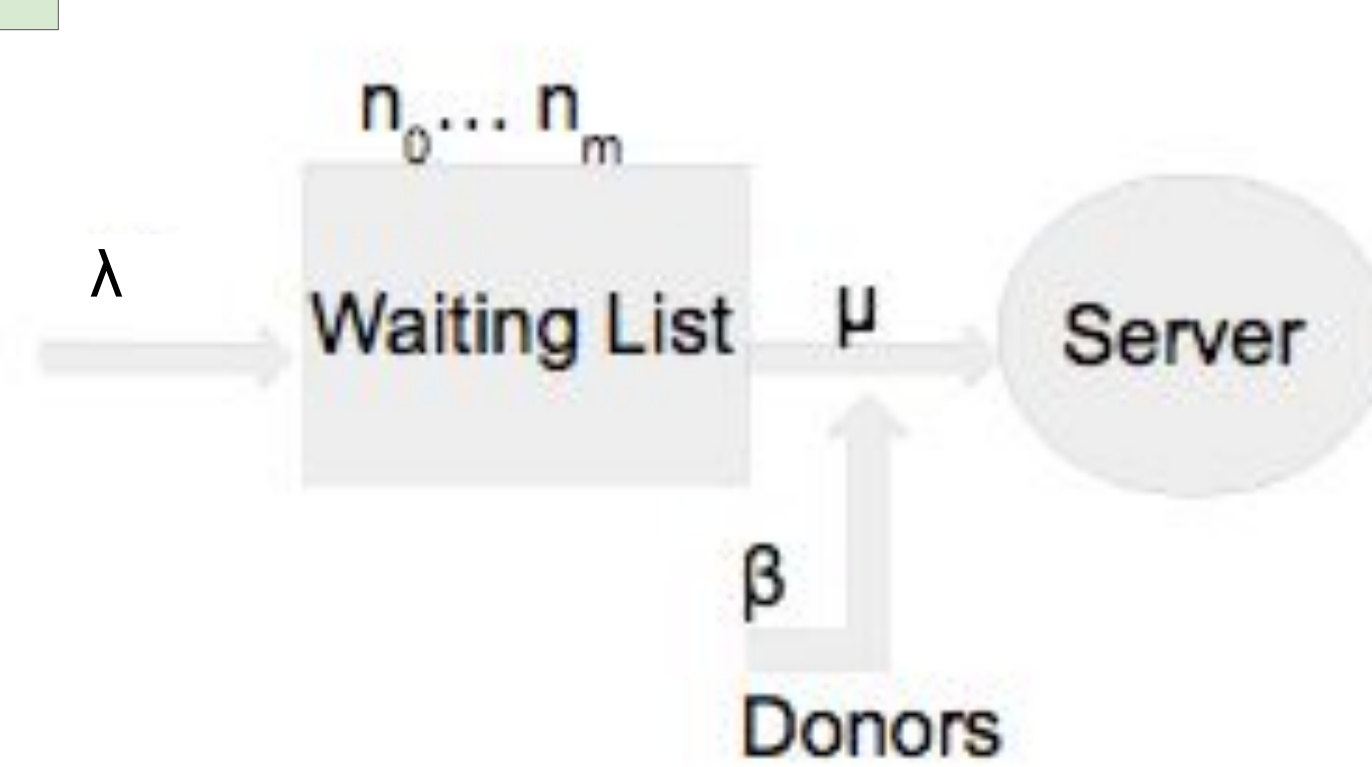
Governing Equations

$$\dot{p}_n(t) = \lambda p_{n-1}(t) - (\lambda + \mu + n\beta)p_n(t) + (\mu + (n+1)\beta)p_{n+1}(t)$$

$$\dot{p}_0(t) = -\lambda p_0(t) + (\mu + \beta)p_1(t)$$

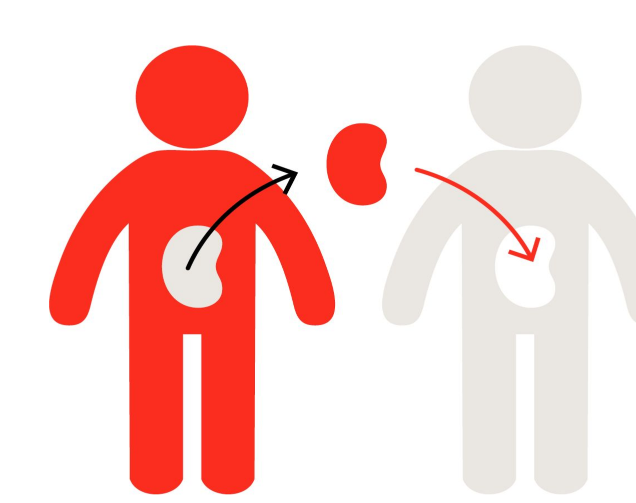
$$P(z,t) = \sum_{n=0} z^n \dot{p}_0(t) \qquad p_n = \frac{\lambda^n 0.2525182297}{\prod_{k=1}^{n}(\mu+k\beta)} \quad n \geq 1$$

$$\frac{dP(z,t)}{dt} = \sum_{n=0}^{\infty} z^n \dot{p}_n(t)$$



Conclusion

The derived differential equations can be plugged into a solver to determine the probability of finding an organ donor for different situations. The use of queueing theory in healthcare has far reaching benefits, as it allows organs to be properly distributed in a more timely manner. Queueing theory is not limited to the healthcare industry, other areas it can be applied to are: traffic flow, project management, industrial engineering, among many others.



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Acknowledgments

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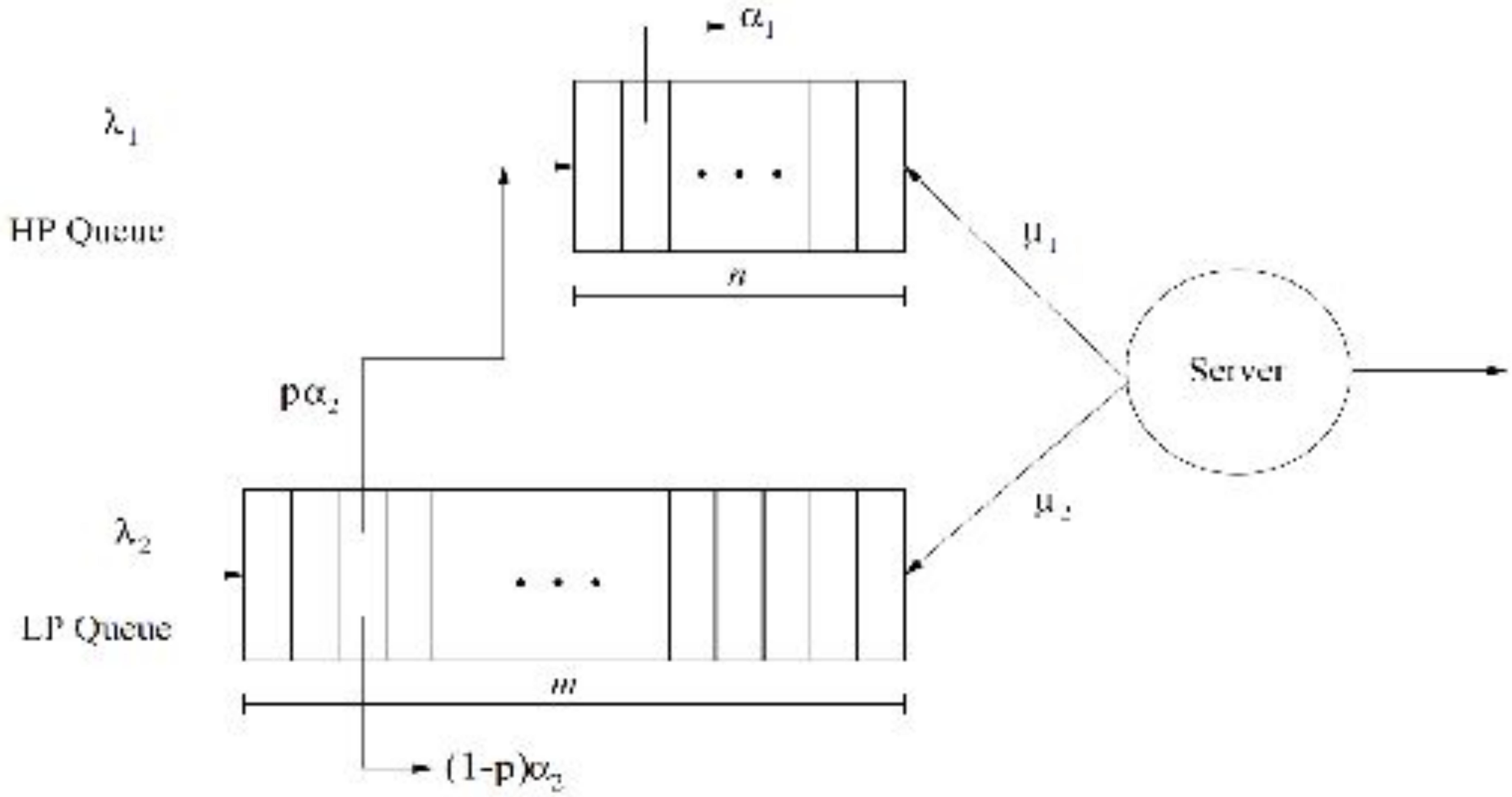
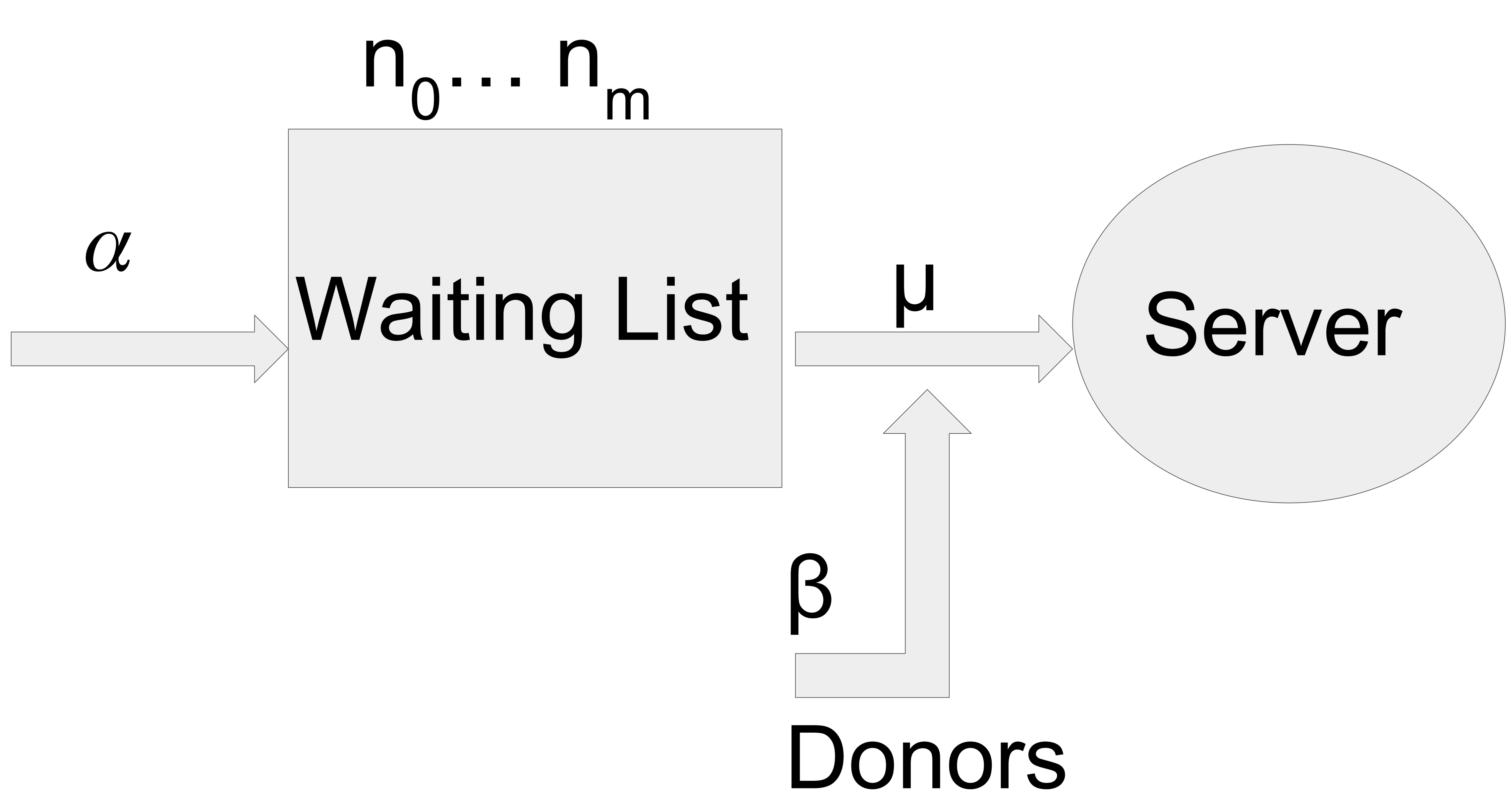
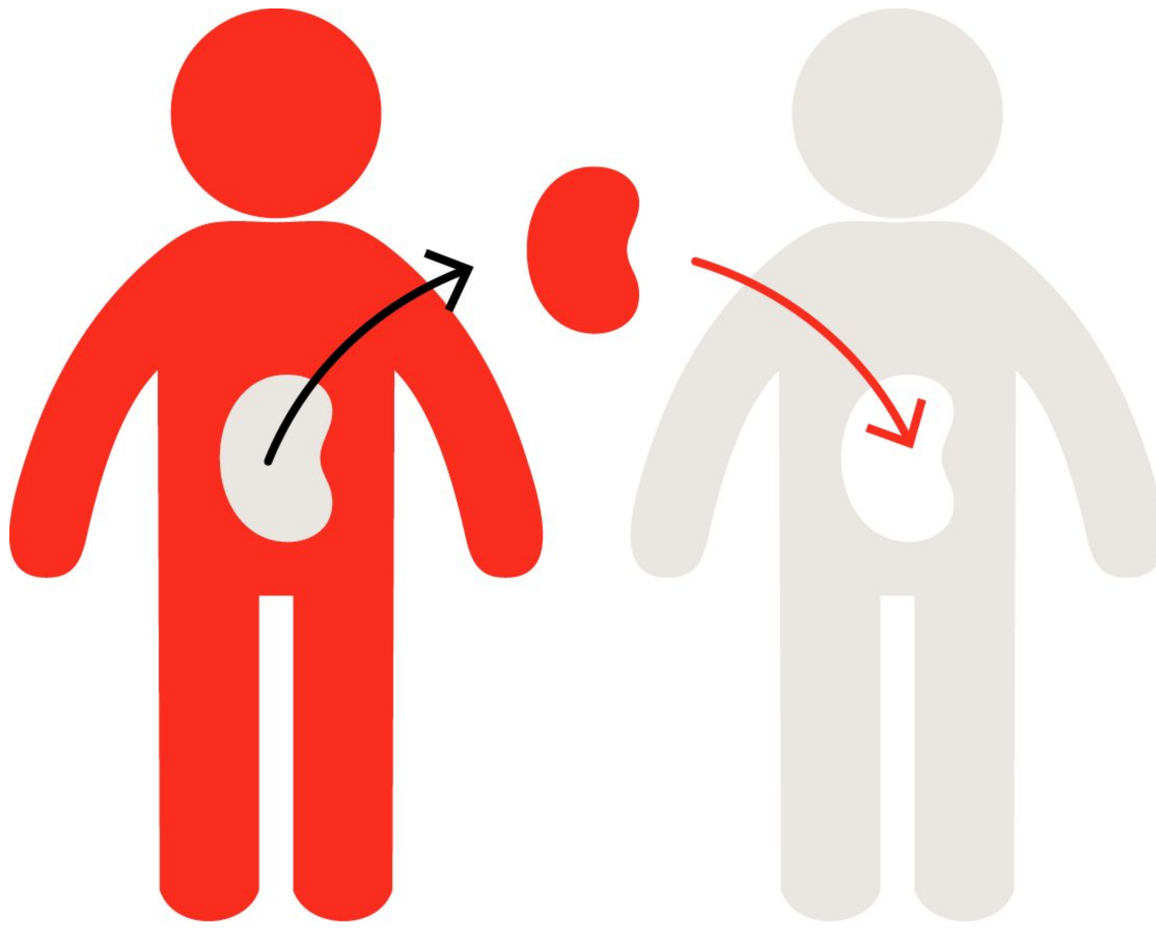


Figure 1: Proposed Queueing Model





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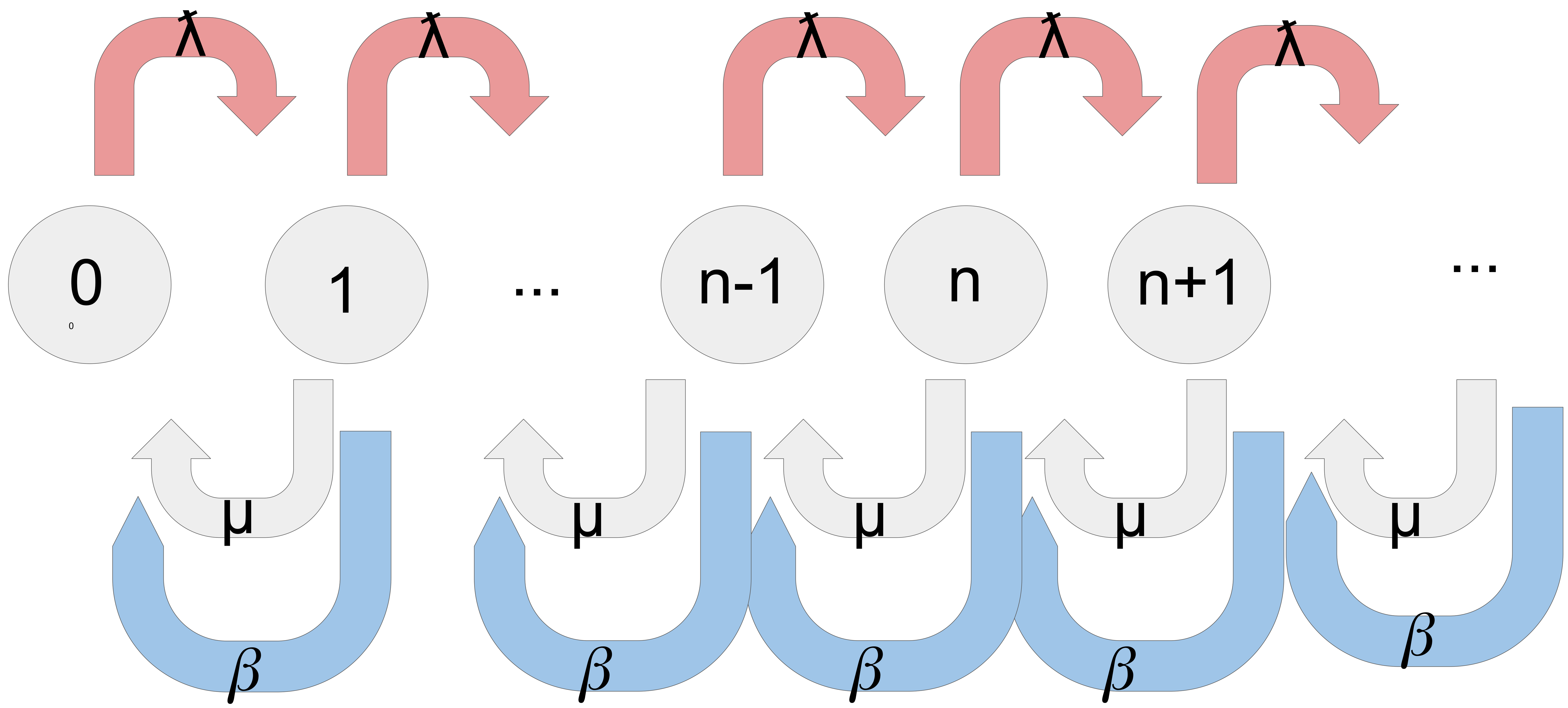
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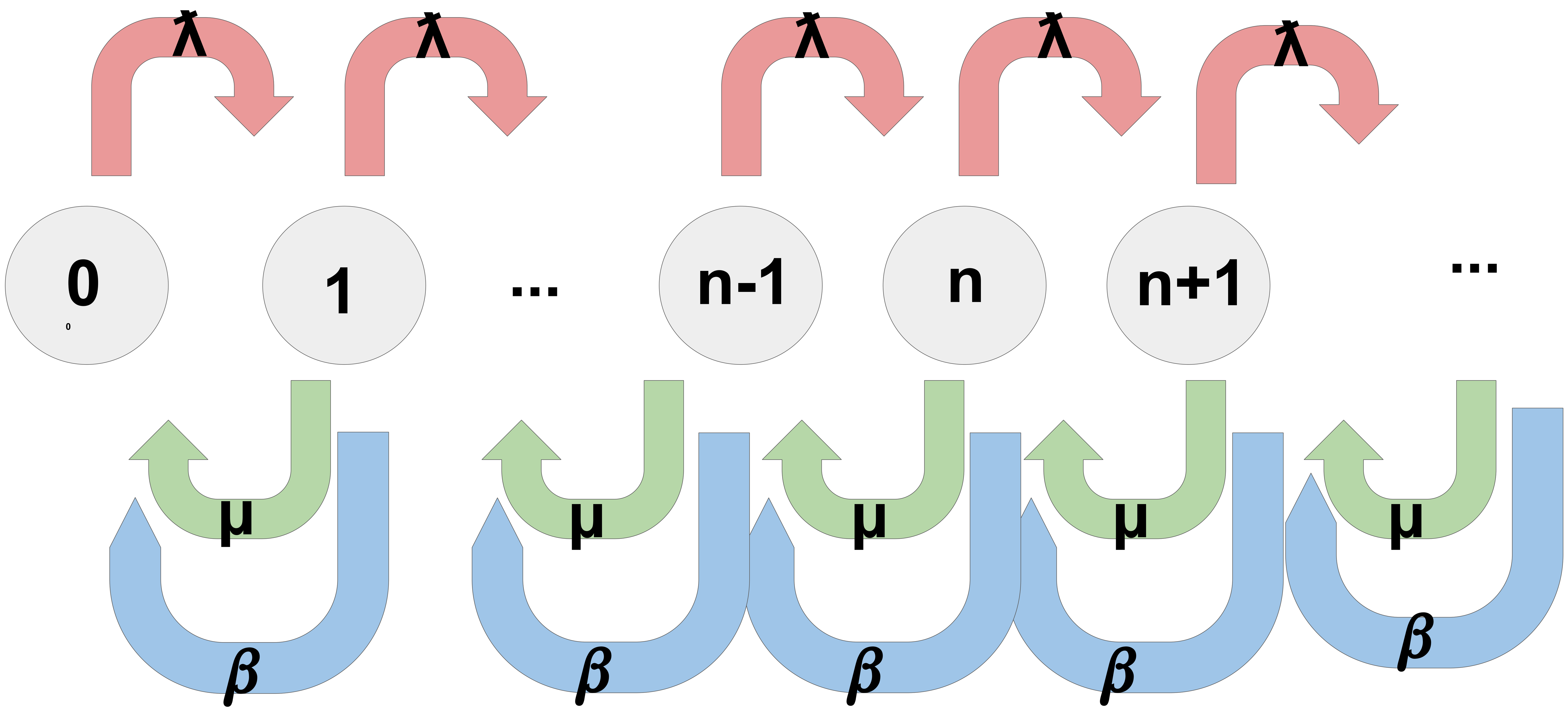

$$\dot{p}_n = \lambda p_{n-1} - (\lambda + \mu + n\beta)p_n + (\mu + (n+1)\beta)p_{n+1}$$

$$\dot{p}_0 = -\lambda p_0 + (\mu + \beta)p_1$$

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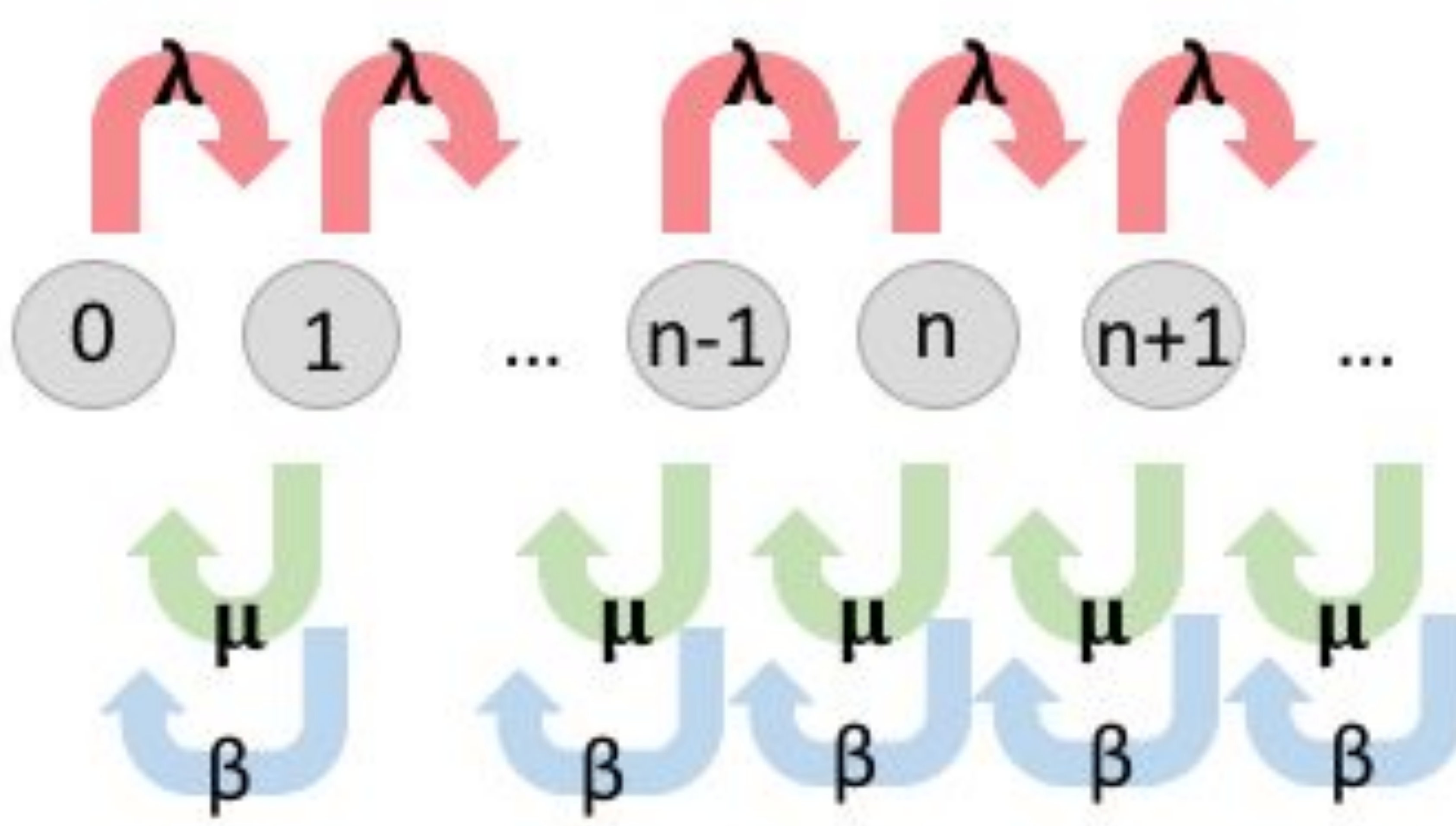

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