Communication Patterns Effect on Privacy

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Introduction

Privacy is important in the world of modern communication. Leakage can occur easily with communication through text messaging and messaging on social media. We will be using graph theory to find the shortest path in messaging from person A to person B, as well as a centralized path from person A to person C and from person C to person B, to figure out which has the least leakage. We will then be trying to minimize the leakage and increase the privacy in the one with the least privacy by using eccentricity to find the central point of the graph.

Methods

Network is modeled as " a graph G = (V , E) consisting of V , a non-empty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints."

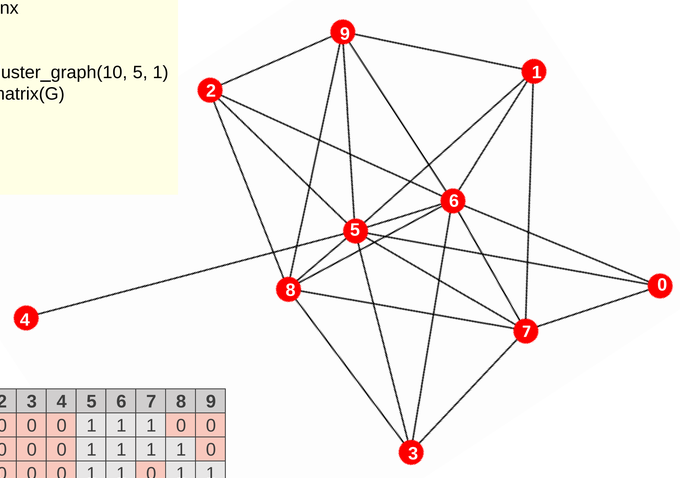
We have created a number graph using Python and networkx, a program through Python that helps with the creation of graphs. The graph consists of 10 nodes, number 0-9.
We then created an adjacency matrix to show to connection between the nodes. 1 means they are connected, 0 means they are not.

```
import sys

import matplotlib.pyplot as plt
import networkx as nx
import scipy as sp

G = nx.powerlaw_cluster_graph(10, 5, 1)
A = nx.adjacency_matrix(G)

nx.draw(G)
print(A.todense())
plt.show()
```



	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	1	1	1	1	0
2	0	0	0	0	0	1	1	0	1	1
3	0	0	0	0	0	1	1	1	0	0
4	0	0	0	0	0	1	0	0	0	0
5	1	1	1	1	1	0	1	1	1	1
6	1	1	1	1	0	1	0	1	1	1
7	1	1	0	1	0	1	1	0	1	0
8	0	1	1	0	0	1	1	1	0	1
9	0	0	1	0	0	1	1	0	1	0

Mesh

Source	Destination	Total Number of Nodes
0	8	3
1	2	3
2	4	3
3	9	3
4	8	3
5	0	2
6	4	3
7	9	3
8	1	3
9	3	3

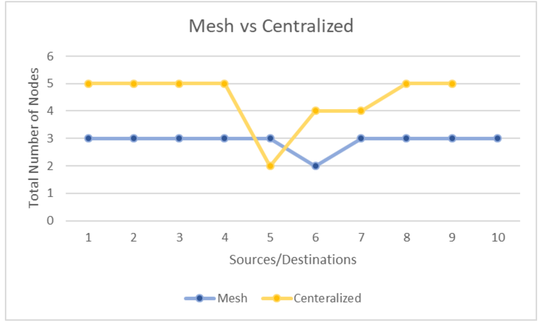
After selecting random destinations for each of the nodes as sources, we found the shortest path to get to each. The calculations for the total number of nodes to receive the "message", on average, was 3 nodes.

Centralized

Source	Center	Destination	Total Number of Nodes
1	0	2	5
2	0	4	5
3	0	9	5
4	0	8	5
5	0	0	2
6	0	4	4
7	0	9	4
8	0	1	5
9	0	3	5

We kept the same sources and destinations, but this time we added a constant centralized node that the "message" would have to pass through before getting to the destination. We excluded the source that was the same at the centralized node. We again found the shortest path. On average, the total number of nodes was 5 nodes.

Conclusion



In conclusion, we have found that having a centralized point will increase the number of nodes who will see the "message," and therefore not be as private as when there is no center point. This will result in more leakage.

Methods to Improve Privacy

The eccentricity of a particular vertex V, in graph G, is the maximum of all the distances from V to any other vertex U in the graph.
$v \epsilon V(g)$ is $e(v)=\max\{d(u,v) \mid u \epsilon V(G)\}$

The diameter of a graph is the maximum eccentricity from vertex V to all other vertex U in the graph.
$diam(G) = \max \{e(v) \mid v \epsilon V(G)\}$.

The radius of a graph is the minimum eccentricity from vertex V to all other vertex U in the graph.
$rad(G) = \min \{e(v) \mid v \epsilon V(G)\}$.

Node	Eccentricity
0	2
1	2
2	2
3	2
4	2
5	1
6	2
7	2
8	2
9	2

The eccentricity for all nodes, except for 5, is 2. The eccentricity of 5 is 1.

The diameter of the graph is 2.
The radius of the graph is 1.

If we make the center of the graph 5, this should improve the privacy and decrease the amount of leakage.

Source	Center	Destination	Total Number of Nodes
0	5	8	3
1	5	2	3
2	5	4	3
3	5	9	3
4	5	8	3
5	5	0	2
6	5	4	3
7	5	9	3
8	5	1	3

By making the center 5, we get the same results for number of nodes for centralized as we do for mesh.

In conclusion, if we use eccentricity, diameter, and radius of a graph, we can find the center node that will minimize the amount of leakage and have the most privacy when sending a message from one node to another.

Resources: Rosen, Kenneth H. Discrete Mathematics and Its Applications. 7th ed., McGraw Hill, 2012.