The Stable Marriage Problem and Gale-Shapley Algorithm

Brock Bavis and Brayton Rider

Abstract

- The stable marriage problem tries to solve stability between two different, but equally sized, sets of data. Stability in this case is defined by a match's members not being able to be better off than the current match. Each element in both sets have a ordered preference list for each element in the other set. The question of whether or not it is possible to create stable marriages between the two sets is answered by the Gale- Shapely algorithm. This algorithm iterates through the different preference lists until a match is found for all elements in the sets. By having one set propose to the other and the other choose over the course of the iterations, the algorithm guarantees that all elements within the sets will be matched and that all marriages will be stable.

Background

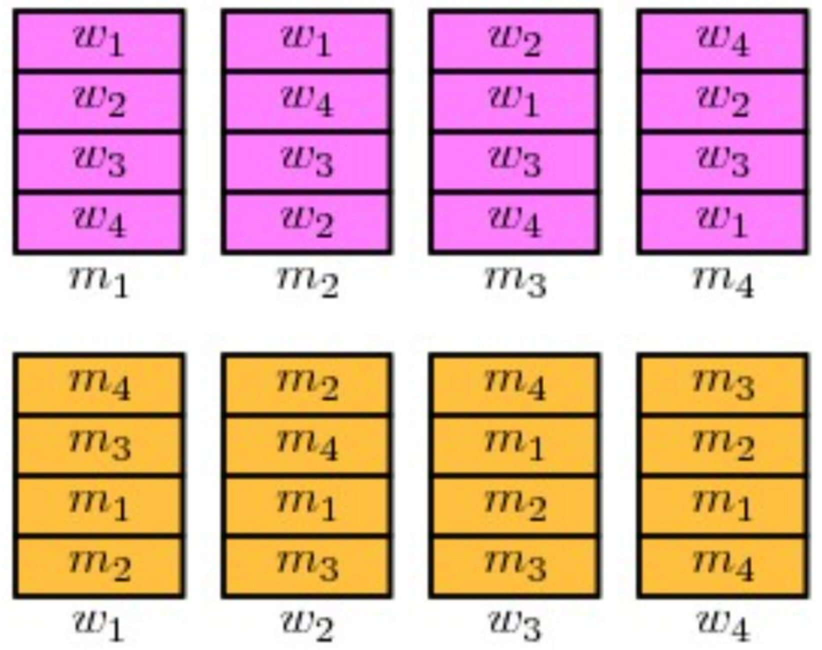
- The Gale-Shapley algorithm was first designed for colleges whom can only accept a certain number of applicants based upon application criteria. The uncertainties of the problem are:
  - if the applicant has applied elsewhere
  - where that particular college ranks on the applicants list of colleges
  - which other colleges accepted that applicant
- Along with the possibility of being put on a "waiting list" for a college, even more problems are introduced which should make a mathematical solution to finding the right college even harder.

Objectives

- The objective of The Stable Marriage Problem and Gale-Shapley algorithm is to match applicants and colleges in the most satisfactory combination possible.
- This desired combination is meant to remove all uncertainties between applicant and college, and to make sure each college fills their quotas for students as closely to their desired amount as possible.

Methods

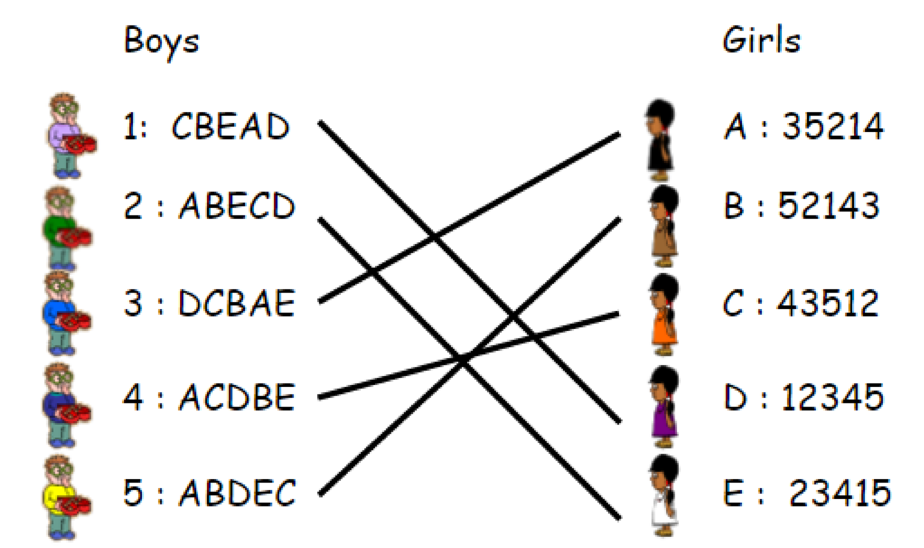
- Step 0:
  - Each element in both sets creates a ranked preference list of all the elements in the other set, (e.g. students chooses a college and colleges choose students)
- Step 1:
  - Each element in the first set (the one deemed as the proposers) proposes to the element at the top of their list
  - Then, the second group rejects everyone who has chosen them, except for the element at the top of their list
- Step 2:
  - Every element in the first set who is not matched proposes to the next element on their preference list
  - The second set can choose to accept the proposal if it is currently unmatched OR it prefers the current proposer rather than the element it is currently matched with, otherwise it can reject the proposal
  - The algorithm continues until every element in the first set finds a match.



Similarity

- The Stable Marriage Problem can be compared to "rush week" for fraternities. Where college fraternities will make bids on those the students' whom the fraternity members deem most fitting for their specific fraternity. They will bid up to their quota limit for new recruits and hope each bid they place will accept. The bid-for students than choose the fraternities with the most attractive offer and the most desirable pairing.

Conclusion



- Many people would assume that there would be a way to "cheat" this system by picking your top choices in strategic order to receive your desirable outcome
- This however, is not the choice. "Rank them in true preference order," was the simple answer by Atila Abdulkadiroglu who worked alongside Roth gave this simple answer whenever anyone would ask for advice for receiving their top choice for any field using a system similar to the outline of the Gale-Shapely paper.

	A	B	C	D
α	1, 3	2, 3	3, 2	4, 3
β	1, 4	4, 1	3, 3	2, 2
γ	2, 2	1, 4	3, 4	4, 1
δ	4, 1	2, 2	3, 1	1, 4

References

- Gale, et al. 1962. "College Admissions and the Stability of Marriage." *The American Mathematical Monthly*. 69(1): 9-15
- Austin, David. "The Stable Marriage Problem and School Choice." *American Mathematical Society.* Web. 4/8/2018

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